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Intermediate Goods and Total Factor Productivity *

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Abstract

The share of intermediate goods in gross output has declined in the U.S. over the 1958-2004 period. I present a model of gross output production in which the intermediate goods share (IGS) in gross output appears as an explicit part of total factor productivity (TFP) in the value added production function. In particular, a larger IGS implies a smaller TFP level. Therefore, the decline in the IGS can contribute to the observed TFP growth in the U.S. during the period considered. A simple growth accounting exercise shows that when the production function for gross output is Cobb-Douglas in capital, labor and intermediate goods, the IGS accounts for at least 1/4 of TFP growth. With a CES gross output production function, the IGS accounts for up to 61% of TFP growth. Using this accounting procedure, I also find that intermediate goods are responsible for the most part of the productivity slowdown occurred during the seventies.

Keywords: Total Factor Productivity Growth, Intermediate Goods, Productivity Slowdown.
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1 Introduction

A central issue in economics is the understanding of the forces driving the growth process of modern economies. For this purpose, the most commonly used tool is the one sector growth model, which is able to replicate the growth process, as observed in the data, as long as one is willing to accept that most of the action is due to an exogenous process for total factor productivity (TFP). Changes in the level of TFP can account for 2/3 of the growth in output per worker in the U.S.[1] and differences in the level of TFP across countries seem to be responsible for the observed differences in output among those countries.[2] These results call for a theory of TFP, as argued in Prescott (1998).[3] The current paper presents a simple theory and strong supporting evidence that suggest that a satisfactory explanation of TFP should take into account intermediate goods in the production process.

Intermediate goods represent an important factor of production in most sectors of industrialized economies.[4] In the U.S., for a given amount of value added, roughly an equivalent amount of intermediate goods is used in the production process at the aggregate level. Despite this fact, common TFP measures are based on a Cobb-Douglas production function in capital and labor only. This procedure is easily justified by the double nature of intermedi-

[1] See Cooley and Prescott (1995).

[2] Prescott (1998).

[3] For recent papers that try to build models that can account for TFP differences see, among others, Parente and Prescott (1999), Herrendorf and Teixeira (2005), Castro, Clementi and MacDonald (2006), Lagos (2006), Guner, Ventura and Xu (2007) and Restuccia and Rogerson (2007).

[4] An intermediate good is represented by any input entering a production process which cannot be described as capital or labor and that depreciates completely during the same production process. Then, intermediate goods include raw materials, energy, components, finished goods and services. That is, intermediate goods are classified by use, as in the input-output tables, and not by type of good.

ate goods, input and output in production, which also suggests that it should be irrelevant whether to consider them or not in aggregate production.[5]

A more careful analysis reveals that excluding intermediate goods is not always a good practice. Suppose production is performed using n types of inputs. Given prices, inputs are used in the optimal proportions. If the price of input i declines relative to the price of the other inputs, given a certain degree of substitutability, the production process becomes more intensive in input i and less in the other inputs. But this implies that the productivity of input i decreases (the quantity of final output over the quantity of input i) and that of the remaining inputs increases. Thus, the productivity of a subset of inputs depends on the utilization of the remaining inputs. This reasoning can be applied to a firm that produces output using capital, labor and intermediate goods. If there is a change in the utilization of intermediate goods, this will affect productivity of the remaining inputs, i.e. capital and labor.

In this paper I construct a model of gross output production that formalizes the intuition given above. The key feature of this framework is that the intermediate goods share in gross output appears as an explicit part of TFP in the value added production function. This occurs because, given a gross output production function in capital, labor and intermediate goods, the extent to which capital and labor contribute to output depends on the quantity

[5] Jorgenson, Gollop and Fraumeni (1987), p. 6, describe their value-added measure in the following way: "*Aggregate output is a function of quantities of sectoral value-added and sums of each type of capital and labor input over all sectors. Deliveries to intermediate demand by all sectors are precisely offset by receipts of intermediate input, so that transactions in intermediate goods do not appear at the aggregate level.*"

and the price of intermediate goods. In particular, a larger intermediate goods share implies a smaller TFP level in the value added production function. I first consider a Cobb-Douglas production function for gross output. With this production function, both the intermediate goods share and the relative price of intermediate goods with respect to gross output appear as explicit parts of TFP. Second, I consider a more general constant elasticity of substitution (CES) production function. With the CES specification, only the intermediate goods share appears as a part of TFP.

I then use this model as a measurement tool, and quantify the influence of intermediate goods on TFP growth in the U.S. To this end, I use two databases. The first is the well-known Dale Jorgenson dataset.[6] The second is the EU KLEMS Database, March 2007 (KLEMS hereafter).[7] The two differ in the time span covered, the former going from 1958 to 1996 and the latter covering the 1970-2004 period. According to these two datasets, the share of intermediate goods in gross output production in the U.S declined by 6.5% between 1958 and 1996, and by 6.6% between 1970 and 2004.[8]

Given the decline in the intermediate goods share, one would expect that, according to the model presented, at least a part of the commonly measured growth in the Solow residual can be explained by the decline in the intermediate goods share of gross output. With the Cobb-Douglas specification, for the 1958-1996 period (Jorgenson dataset), the change in the

[6] Downloadable at http://post.economics.harvard.edu/faculty/jorgenson/data/35klem.html.

[7] Downloadable at http://www.euklems.net/

[8] Gross output is defined as the sum of value added and intermediate goods.

intermediate goods share can account for 27% of TFP growth in the U.S., if the relative price of intermediate goods with respect to gross output is assumed to be constant over time. If instead I take into account the observed change in this relative price over the period considered, the change in the intermediate goods share and the relative price of intermediate goods can jointly account for 25% of TFP growth. The corresponding results for the period 1970-2004 (KLEMS dataset) are 29% and 45%, respectively.

Next, the same numerical exercise is repeated with a more general CES production function in intermediate goods and value added. This is done to account for the findings of Bruno (1984) and Rotemberg and Woodford (1996), who estimate an elasticity of substitution between value added and intermediate goods smaller than one. In the CES case, the value of the elasticity of substitution between value added and intermediate goods becomes crucial in determining the impact of changes in the share of intermediate goods on TFP growth. I use the values estimated in Bruno (1984) and Rotemberg and Woodford (1995) and find that the decrease in the share of intermediate goods is, as in the Cobb-Douglas model, a source of observed TFP growth. The decline in the intermediate goods share can account for up to 61% of TFP growth. I also estimate, using the KLEMS dataset, a common elasticity of substitution between value added and intermediate goods across sectors. The estimated value of the elasticity of substitution is 0.14, which implies that the contribution of the share of intermediate goods is 19% of TFP growth.

The importance of the intermediate goods share in TFP is also highlighted when I measure how much the change in the intermediate goods share accounts for the *productivity slowdown* observed in the seventies. The intermediate goods share, although declining over the entire time-span considered, increases remarkably during that period. This increase is able to explain most of the productivity slowdown across specifications. In some cases (depending on the parameters of the gross output production function), the increase in the share of intermediate goods accounts for more than the observed slowdown in the Solow residual. The implication of this fact is that exogenous productivity might, in fact, have been increasing during the "slowdown period".

In the last part of the paper I perform two decompositions. These are done to shed some light on the reasons underlying the decrease in the intermediate goods share. For each sector, the latter can be written as the product of two components, the ratio of the value of intermediate goods used in that sector to the value of gross output in the same sector, and the ratio of the value of gross output in that sector to the total value of gross output in the economy. The first ratio is a measure of the intermediate goods share within the sector. The second ratio represents the weight of the sector in the economy. Keeping fixed one of the two components at the 1958 level, provides information on the contribution of the other component to movements in the share of intermediate goods. In the first decomposition the weights of the various sectors in the economy are fixed to their 1958 level and the resulting time series for the intermediate goods share is computed. In the second I fix the intermediate goods share in each sector at its 1958 level and compute again the intermediate goods share.

These two experiments suggest that the long run decrease of the share is due to a change in the weights of the different sectors in the economy. The peak occurred during the seventies, instead, reflects an increase in the share of intermediate goods in each sector during that period.

This is not the first work stressing the importance of intermediate goods for productivity measures. Bruno (1984) shows that an increase in the price of intermediate goods used in a given sector is equivalent to a Hicks-neutral negative technological shock in the value added production function of that sector. His analysis can be extended to show that not only the price of intermediate goods is relevant for value added productivity but also their utilization. This analysis is carried out in Baily (1986). Both Bruno (1984) and Baily (1986) focus their empirical analysis on the manufacturing sector while the current work focuses on the U.S. economy as a whole.

The paper is organized as follows: section 2 presents a model of gross output production that uses a Cobb-Douglas production function for gross output; using this model, section 3 provides a quantitative analysis of the relevance of intermediate goods for TFP measures; section 4 generalizes the analysis to the CES case; section 5 discusses the quantitative relevance of the intermediate goods share for the productivity slowdown; section 6 presents the decompositions of the intermediate goods share; section 7 concludes.

2 A simple model

Consider a production economy in which intermediate goods M are used as inputs in the production process, with capital K and labor N, through an aggregate production function $G(A, K, N, M)$. Here I will derive a standard value added production function starting from a more general problem of gross output production.

The representative firm solves[9]

$$\max_{K,N,M} \{G(A, K, N, M) - rK - wN - pM\}. \tag{1}$$

Here $G(A, K, N, M)$ is a constant returns to scale gross output production function in the three inputs, and a Hicks-neutral productivity level A. Markets are competitive so the firm takes the price of capital r, that of labor w, and that of intermediate goods p as given.[10]

I consider a Cobb-Douglas production function for gross output

$$G(A, K, N, M) = AK^{\alpha}N^{\beta}M^{1-\alpha-\beta}, \tag{2}$$

with α, $\beta > 0$ and $\alpha + \beta < 1$, as this specification allows me to control the shares of the various factors in gross output by looking at the parameter values.[11]

[9] For the motivation of this type of problem see also Rotemberg and Woodford (1995).

[10] Some authors, Rotemberg and Woodford (1995) for instance, consider the relative price of intermediate goods with respect to gross output equal to one. However, as stressed in Bruno (1984), movements in the relative price of intermediate goods can affect value added productivity. As my main concern here is to measure the importance of intermediate goods for TFP I allow p to vary in the theoretical model. In the empirical analysis I then consider both the case in which p is constant and the case in which it is free to vary.

[11] I could have instead considered a nested version of the production function in (2)

$$G(K, N, M) = \left(K^{\theta}N^{1-\theta}\right)^{\eta} M^{1-\eta},$$

with η and θ between zero and one. However, both specifications of the production function deliver the

Using (2), the first order condition for the problem (1) with respect to intermediate goods implies that, at the optimal solution,

$$M^* = (1-\alpha-\beta)^{\frac{1}{\alpha+\beta}} p^{-\frac{1}{\alpha+\beta}} A^{\frac{1}{\alpha+\beta}} K^{\frac{\alpha}{\alpha+\beta}} N^{\frac{\beta}{\alpha+\beta}}. \tag{3}$$

Using (3) in problem (1) I obtain

$$\max_{K,N} \left\{ (\alpha+\beta)(1-\alpha-\beta)^{\frac{1-\alpha-\beta}{\alpha+\beta}} p^{1-\frac{1}{\alpha+\beta}} A^{\frac{1}{\alpha+\beta}} K^{\frac{\alpha}{\alpha+\beta}} N^{\frac{\beta}{\alpha+\beta}} - rK - wN \right\},$$

which can be rewritten as

$$\max_{K,N} \left\{ A^{\frac{1}{\alpha+\beta}} B K^{\gamma} N^{1-\gamma} - rK - wN \right\}, \tag{4}$$

where $\gamma = \frac{\alpha}{\alpha+\beta}$, $1-\gamma = \frac{\beta}{\alpha+\beta}$, and

$$B = (\alpha+\beta)(1-\alpha-\beta)^{\frac{1-\alpha-\beta}{\alpha+\beta}} p^{1-\frac{1}{\alpha+\beta}}. \tag{5}$$

With this formulation, I define

$$Y = A^{\frac{1}{\alpha+\beta}} B K^{\gamma} N^{1-\gamma}, \tag{6}$$

which is a constant returns to scale Cobb-Douglas production function in capital and labor. The function Y is interpreted as the value added production function in two inputs, capital and labor, and a standard productivity term C

$$Y = CK^{\gamma} N^{1-\gamma}, \tag{7}$$

same optimal solution to (1). This is evident once one defines the equivalences $\theta = \frac{\alpha}{\alpha+\beta}$, $1-\theta = \frac{\beta}{\alpha+\beta}$ and $\eta = \alpha+\beta$.

where $C = A^{\frac{1}{\alpha+\beta}} B$.[12]

The maximization problem (4) is a standard two-input problem as constructed in growth and business cycle theory. Thus, starting from a more general specification of gross production it is always possible to derive the standard problem. This procedure has the nice feature of providing additional information on the TFP term in (7), C. The latter can in fact be decomposed into $A^{\frac{1}{\alpha+\beta}}$ and B. In particular, B depends only on the share of intermediate goods, $1-\alpha-\beta$, and the price of intermediate goods in terms of gross output, p. Then, a natural question is how much $A^{\frac{1}{\alpha+\beta}}$ and B contribute to C.

As in Bruno (1984), an increase in the price of intermediate goods is equivalent to a Hicks-neutral negative shock on the value added production function. In particular, given (5), an increase in p affects negatively B. In addition, the share of intermediate goods becomes a crucial variable in determining the level of TFP. A larger intermediate goods share in (5), $1-\alpha-\beta$, implies a smaller B. A part of the commonly measured change in the Solow residual can now be explained by using intermediate goods, thus reducing the importance of unexplained productivity A.

As the next section will show, the share of intermediate goods declined during the 1958-2004 period in the U.S. According to the model presented, at least part of the change in value added TFP can be accounted for by the decline in the intermediate goods share. The

[12] In defining (6) I implicitly assume that the price of value added in terms of gross output is equal to one. To relax this assumption I should define $p_y Y = A^{\frac{1}{\alpha+\beta}} B K^{\gamma} N^{1-\gamma}$ with p_y being that price. In this case, the TFP measure for value added Y would be $A^{\frac{1}{\alpha+\beta}} B/p_y$. As the focus of the paper is to measure the contribution of the intermediate goods share to TFP growth, reflected in the term B, I consider $p_y = 1$. In this way I implicitly assume that the growth of $A^{\frac{1}{\alpha+\beta}}$ includes also the effect due to the growth in p_y.

aim of the following section is therefore to assess the quantitative relevance of the change in the intermediate goods share for TFP growth.

3 Quantitative Results

In this section, I use Dale Jorgenson and KLEMS datasets to present evidence on the behavior of the intermediate goods share in the U.S. over the period 1958-2004 and its contribution to TFP growth. I proceed as follows. First, using standard growth accounting methodology I calculate C, which appears in (7), using data on capital and labor inputs, capital and labor shares and real GDP.[13] Next, I calculate the empirical counterpart to B, as reported in (5). In order to do this, I follow the input-output tables definition of gross output, and define the latter as

$$G_t = p_t^y Y_t + p_t M_t, \tag{8}$$

where Y_t and M_t are indices of value added and intermediate goods in the economy, p_t is the relative price of intermediate goods with respect to gross output and p_t^y the relative price of value added. Dividing equation (8) by G_t, I obtain

$$1 = \frac{p_t^y Y_t}{G_t} + \frac{p_t M_t}{G_t}. \tag{9}$$

I define $p_t M_t / G_t$ the intermediate goods share of gross output (IGS hereafter).

Figure 1 reports the IGS for the U.S. economy computed using Jorgenson and KLEMS

[13] Note that, in equation (7), γ depends on the intermediate goods share through α and β. However, a constant γ, as observed in the data, is consistent with a variable intermediate goods share, as long as α and β change at the same rate.

datasets. I also report the trend of both series, computed using the Hodrick-Prescott (HP) filter. In both cases, the IGS declines over time and there is a temporary increase during

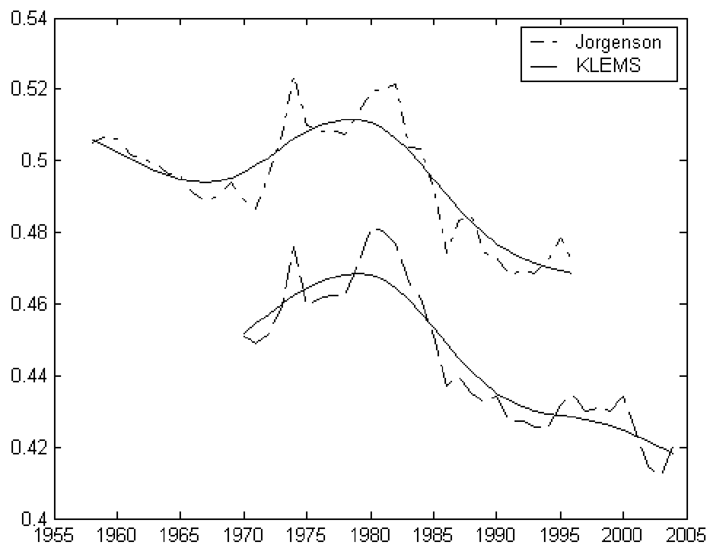

Figure 1: Intermediate Goods Share of Gross Output in the U.S. for the period 1956-1996 (Jorgenson) and for the period 1970-2004 (KLEMS). Source: D. Jorgenson web page, EU KLEMS Database, March 2007, and own calculation.

the period going from 1973 to 1982, roughly. The different levels of the two series are due to different normalizations of the price indices used. This is not a concern here, as the relevant feature for the calculations performed in this work is the time-pattern of the IGS and not its absolute level.[14]

As my aim is to assess the quantitative relevance that changes in the IGS and in p had

[14] I also check stationarity of the IGS time series using Jorgenson dataset. I first regress the IGS over its first lag and a constant. The constant term is non significant. Next, I run a regression of the IGS over its first lag only. The Dickey-Fuller test leads to accept the null hypothesis of a unit root. Finally, I regress the IGS over a constant and a time trend. Both coefficients are significant and the time trend, although small, is negative. Econometric results suggest that the IGS is declining over time.

for TFP during the period considered, I calculate the empirical equivalent to (5) as

$$B_t = (1 - IGS_t)\, IGS_t^{\frac{IGS_t}{1-IGS_t}} p_t^{-\frac{IGS_t}{1-IGS_t}}. \tag{10}$$

In order to compute (10), I need a measure for p_t. As this measure might be sensitive to the price index chosen, I present two different specifications: the benchmark case in which p_t is constant over time and the case in which p_t is free to vary. I then compare the growth rate of B_t with the growth rate of the Solow residual C_t.

The yearly average growth rates for C_t and B_t are reported in Table 1. The numbers in parenthesis in each table represent the corresponding results obtained with Hodrick-Prescott trend series. In the first column of Table 1 I report the Solow residual average growth rate.[15] The details of calculations are reported in Appendix B.

Table 1 (Total Factor Productivity Growth)

Cobb-Douglas Case with a constant Intermediate Goods Price

	C (Solow Residual)	*B* (*p=1*)	Ratio (2)/(1)
Jorgenson (1958-1996)	0.89% (0.87%)	0.24% (0.27%)	0.27 (0.31)
KLEMS (1970-2004)	0.80% (0.70%)	0.23% (0.26%)	0.29 (0.37)
Jorgenson (1970-1996)	0.60% (0.56%)	0.17% (0.29%)	0.28 (0.52)
KLEMS (1970-1996)	0.69% (0.62%)	0.16% (0.24%)	0.23 (0.39)

The first column of the table reports the average growth rate for the Solow residual in the U.S., *C*. The second column reports *B* for a constant intermediate goods relative price. The third column is the ratio of the second to the first column.
Results for HP trend series are reported in parenthesis.
Source: D. Jorgenson webpage, KLEMS dataset and own calculation.

The Solow residual displays an average growth rate of 0.89% per year during the 1958-1996 period and 0.80% during the 1970-2004 period. The corresponding values for B_t are 0.24%

[15] The growth rate of the Solow residual is in line with other works. See for instance Wolff (1996) for measures covering roughly the same time period as Jorgenson dataset.

and 0.23% respectively. These values imply a remarkable growth contribution of the IGS to the Solow residual of 0.27 and 0.29 during the two periods. A higher contribution in the 1970-2004 time span is confirmed when using HP trend series, 0.31 and 0.37 respectively.

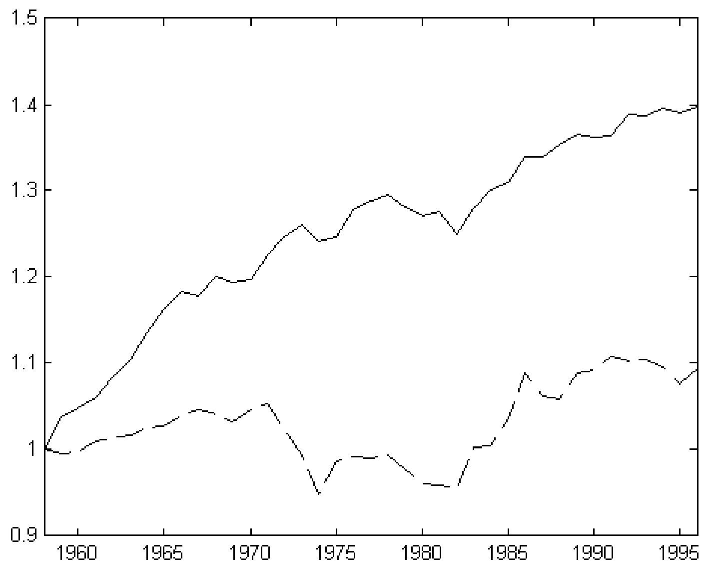

Figure 2: The continuous line represents the Solow residual and the dashed line represents B, the part of the Solow residual due to the intermediate goods share, for the 1958-1996 period. Series are normalized to be one in 1958. Source: Jorgenson dataset, the Bureau of Economic Analysis and own calculation.

I also perform the calculation for the time period covered in both datasets, 1970-1996. In this case, it can be observed that the contribution of B_t to C_t is slightly larger with Jorgenson dataset, 0.28 versus 0.23. The insight that Table 1 provides is that the part of TFP due to the change in the intermediate goods share, B_t, accounts for roughly one fourth of the growth in the Solow residual.

The series for B_t with constant p_t, and the Benchmark Solow residual C_t are reported in Figure 2 and 3 for the two datasets, where they have been normalized to one in the first period of the sample. Some remarks can be made about these two pictures. First, during the so-called productivity slowdown, starting at the beginning of the seventies and lasting until the first years of the eighties, B_t displayed negative growth. Subsequently, the recovery of the Solow residual between 1982 and 1985 is accompanied by a sustained increase in B_t which begins one or two years earlier. Finally, in Figure 3, the increase in the growth of C_t from 2001 is accompanied by an increase in B_t, starting in 2000.

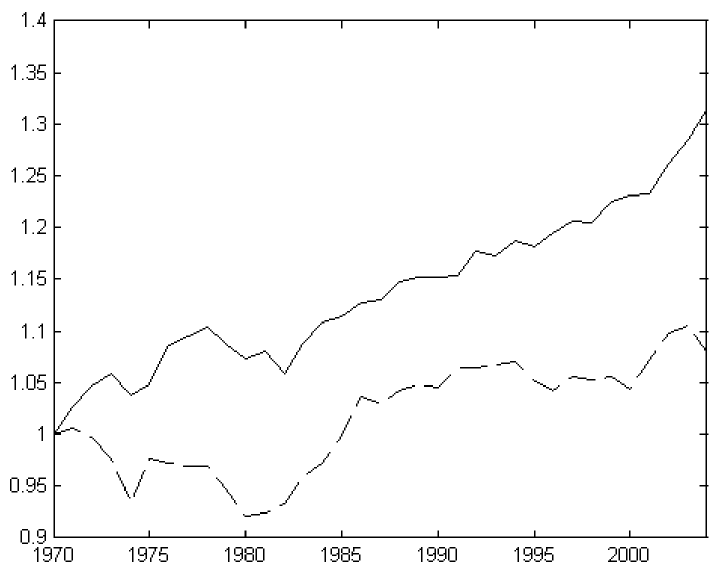

Figure 3: The continuous line represents the Solow residual and the dashed line represents B, the part of the Solow residual due to the intermediate goods share, for the 1970-2004 period. Series are normalized to be one in 1970. Source: EU KLEMS Database, March 2007, the Bureau of Economic Analysis and own calculation.

I now take into account movements in the price of intermediate goods in calculating B_t. The second column of Table 2 reports the measure for B_t where the relative price p_t is variable in formula (10). With Jorgenson's dataset, I construct p_t as a Fisher index. The methodology adopted to compute the price index is reported in Appendix B. The KLEMS database reports prices for aggregate gross output and intermediate goods. I use these prices to compute the relative price of intermediate goods used in the calculations involving the KLEMS dataset.

The growth rate of B_t is slightly smaller with respect to the case with a constant p_t (0.22% as opposed to 0.24%) when I use Jorgenson's data. The ratio of B_t and the Solow residual growth rate is 0.25. Things are different with the KLEMS dataset. In this case B_t accounts for 0.45 of the growth in C_t. The result is remarkable because it implies that B_t accounts for almost half of the growth in C_t. Clearly, the difference between Table 1 and 2 lies in the relative price of intermediate goods.

Table 2 (Total Factor Productivity Growth)

Cobb-Douglas Case with a variable Intermediate Goods Price

	C (Solow Residual)	*B* (variable *p*)	Ratio (2)/(1)
Jorgenson (1958-1996)	0.89%	0.22%	0.25
	(0.87%)	(0.27%)	(0.31)
KLEMS (1970-2004)	0.80%	0.36%	0.45
	(0.70%)	(0.38%)	(0.54)
Jorgenson (1970-1996)	0.60%	0.09%	0.15
	(0.56%)	(0.25%)	(0.45)
KLEMS (1970-1996)	0.69%	0.21%	0.30
	(0.62%)	(0.28%)	(0.45)

The first column of the table reports the average growth rate for the Solow residual in the U.S., *C*. The second column reports *B* for a variable intermediate goods relative price. The third column is the ratio of the second to the first column.
Results for HP trend series are reported in parenthesis.
Source: D. Jorgenson webpage, KLEMS dataset and own calculation.

Figure 4 reports the relative price of intermediate goods for the two datasets, where both series are normalized to one in the first period of the sample. This price peaks during the

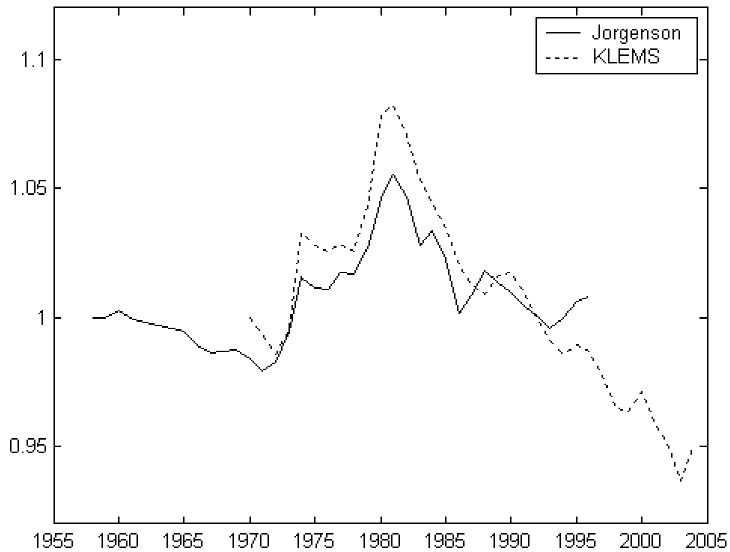

Figure 4: Time pattern of the relative price of intermediate goods in terms of gross output. Source: D. Jorgenson webpage, EU KLEMS Database, March 2007 and own calculation.

seventies and the mid-eighties, but returns roughly to its 1958 level at the end of the sample in Jorgenson's dataset, being 1.007 in 1996. This is the reason why a constant or variable price does not make a dramatic difference when performing calculations regarding the 1958-1996 period. However, as the KLEMS dataset shows, the relative price of intermediate goods has declined since 1982 and this has a non-negligible effect on the Solow residual growth.

Finally, it should be noted that there is a difference between the two datasets in the contribution of B_t to C_t during the common subperiod 1970-1996, 0.15 versus 0.30. However, this difference disappears with HP trend series. This happens because the average growth rate of each series depends on the initial and final level of the series. It follows that average growth rates might depend crucially, with a short dataset and yearly data, on a particular value of the series in the initial or the final year. In such cases it is useful to look at HP trend series instead of the original series.

I conclude that, under the assumption of a Cobb-Douglas production technology for gross output, changes in the IGS can account for a relevant part of the Solow residual growth during the period considered.

4 The CES case

The Cobb-Douglas specification presented in the previous section represents a straightforward tool to measure the influence of the IGS on TFP. However, some studies suggest that the elasticity of substitution between value added and intermediate goods is less than unity. Bruno (1984) suggests a lower bound of 0.3 across industrialized countries in the manufacturing sector, reporting a value of 0.45 for the U.S. while Rotemberg and Woodford (1996) report instead a value of 0.686. Rotemberg and Woodford (1996) estimate the elasticity of substitution between value added and intermediate goods, under the assumption of perfect competition, using data for 20 two-digit U.S. manufacturing sectors supplied by the BLS Division of Productivity Research.

To take into account an elasticity of substitution smaller than one, I repeat the quantitative exercise proposed in the last section employing a more general CES production function in the gross output maximization problem. The latter becomes

$$\max_{K,N,M} \left\{ A \left[\theta M^{\sigma} + (1-\theta) \left(K^{\gamma} N^{1-\gamma} \right)^{\sigma} \right]^{1/\sigma} - pM - rK - wN \right\}, \tag{11}$$

where $G = A\left[\theta M^{\sigma} + (1-\theta)\left(K^{\gamma}N^{1-\gamma}\right)^{\sigma}\right]^{1/\sigma}$ is the gross output production function. Here σ represents the parameter governing the elasticity of substitution between intermediate goods and capital and labor, $1/(1-\sigma)$, these latter nested in a standard constant returns to scale Cobb-Douglas production function with parameter γ. All remaining variables are as previously defined. Appendix C shows that, using the first order condition with respect to M, the problem can be written in a reduced form as

$$\max_{K,N}\left\{\left[1-\frac{pM}{G}\right]^{1-1/\sigma} A\,(1-\theta)^{1/\sigma} K^{\gamma}N^{1-\gamma} - rK - wN\right\}. \tag{12}$$

I define value added as

$$Y = AB\,(1-\theta)^{1/\sigma} K^{\gamma}N^{1-\gamma}. \tag{13}$$

where $B = [1 - pM/G]^{1-1/\sigma}$. As a result, TFP in the value added production function (13), defined as $C = AB\,(1-\theta)^{1/\sigma}$ depends on unexplained productivity, A, and on the share of intermediate goods in gross output, pM/G, which is a part of B.[16] As in the previous section, the production function for value added can be expressed as a constant returns to scale production function in capital and labor with TFP level C.

About (12), an additional remark is noteworthy. Suppose that the Cobb-Douglas aggregator in (11), $K^{\gamma}N^{1-\gamma}$, possesses its own exogenous productivity level, D. Then, (11) would

[16]Note that the term $[1 - pM/G]^{1-1/\sigma}$ depends, through the first order conditions, on the levels of K and N that are chosen in equilibrium. Then, a part of value added TFP is endogenous and it is determined by choosing the optimal K and N. In equilibrium, once K and N are found, value added TFP is given by the whole term $[1 - pM/G]^{1-1/\sigma} A\,(1-\theta)^{1/\sigma}$ and $[1 - pM/G]^{1-1/\sigma}$ depends on the realized share of intermediate goods.

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$$\max_{K,N,M} \left\{ A \left[\theta M^{\sigma} + (1-\theta) \left(D K^{\gamma} N^{1-\gamma} \right)^{\sigma} \right]^{1/\sigma} - pM - rK - wN \right\}.$$

In this case, the reduced form problem is

$$\max_{K,N} \left\{ \left[1 - \frac{pM}{G} \right]^{1-1/\sigma} AD \left(1-\theta\right)^{1/\sigma} K^{\gamma} N^{1-\gamma} - rK - wN \right\}.$$

The last expression shows that the effect of exogenous productivity on gross output and on value added act both as a Hicks-neutral productivity level on the reduced form Cobb-Douglas aggregator of capital and labor. The same reasoning applies if one assumes labor augmenting exogenous productivity. Then, the term A in (13) can be interpreted as the overall effect of all types of exogenous productivity on the Solow residual. The remaining part is clearly explained by the term B in (13).

My strategy is again to compare the growth rate of C with that of B, the latter calculated using data on the IGS. With respect to the Cobb-Douglas case, this specification permits to avoid the calculation of a price index for intermediate goods. On the other hand, to assess the contribution of the IGS to the Solow residual, I need to choose the parameter σ governing the elasticity of substitution between value added and intermediate goods. This value becomes important because the contribution of the IGS to total TFP growth is now given by the growth rate of $1-pM/G$ multiplied by $1-1/\sigma$. It is clear that this specification implies that a decrease in the IGS implies an increase in the Solow residual only if $\sigma < 0$. I perform the calculations using two values of the elasticity of substitution, $1/(1-\sigma)$, taken form the literature: 0.45, as reported by Bruno (1984) and 0.686, as estimated by Rotemberg and Woodford (1996). The corresponding values for σ are -1.22 and -0.46, respectively. In addition, I estimate a common elasticity of substitution across sectors using the KLEMS dataset. This dataset provides series of indices of quantity and price for value added and intermediate goods across sectors over the 1970-2004 period. I use a pooled ordinary least square to estimate a common elasticity of substitution across sectors over the period considered. In Appendix D I report the methodology adopted. The estimate obtained is 0.14, which implies $\sigma = -6.18$. Table 3 reports the average growth rate of the $[1 - pM/G]^{1-1/\sigma}$ term for the three values of σ.

Table 3 (Total Factor Productivity Growth)

CES Case

	C (Solow Residual)	*B* (sigma=-6.18)	Ratio (2)/(1)	*B* (sigma=-1.22)	Ratio (4)/(1)	*B* (sigma=-0.46)	Ratio (6)/(1)
Jorgenson (1958-1996)	0.89%	0.20%	0.22	0.31%	0.35	0.54%	0.61
	(0.87%)	(0.22%)	(0.25)	(0.35%)	(0.40)	(0.61%)	(0.70)
KLEMS (1970-2004)	0.80%	0.18%	0.23	0.28%	0.35	0.49%	0.61
	(0.70%)	(0.20%)	(0.29)	(0.32%)	(0.46)	(0.56%)	(0.80)
Jorgenson (1970-1996)	0.60%	0.14%	0.23	0.22%	0.37	0.39%	0.65
	(0.56%)	(0.24%)	(0.43)	(0.38%)	(0.68)	(0.66%)	(1.18)
KLEMS (1970-1996)	0.69%	0.13%	0.19	0.20%	0.29	0.35%	0.51
	(0.62%)	(0.19%)	(0.31)	(0.30%)	(0.48)	(0.52%)	(0.84)

The first column of the table reports the average growth rate for the Solow residual in the U.S, *C*. The second column reports *B* from the CES case calculated using sigma=-6.18 (KLEMS), the fourth using sigma=-1.22 (Bruno) and the sixth that calculated using sigma=-0.46 (Rotemberg and Woodford). The third column is the ratio of the second to the first column, the fifth column the ratio of the fourth to the first column and the seventh column the ratio of the sixth to the first column. Results for HP trend series are reported in parenthesis.
Results for HP trend series are reported in parenthesis.
Source: D. Jorgenson webpage, KLEMS dataset and my calculation.

The importance of the IGS is roughly in line with the Cobb-Douglas case when I consider the value of σ suggested by Bruno. Consider the fifth column of Table 3. Given an elasticity of substitution of 0.45, changes in the IGS explain up to 0.37 of total TFP growth. If I consider instead a higher value for the elasticity of substitution, 0.686, as estimated by Rotemberg and Woodford (1996), the IGS explains at least 0.51 of TFP growth. With this elasticity of substitution then, more than half of TFP growth would be explained by the IGS variation. The contribution of B is lower when I consider the value of the elasticity of substitution estimated from the KLEMS dataset. It must be pointed out that the quantitative results are quite sensitive to the parameter σ. However, the results presented apply for a wide range of values of σ. The lower bound for the contribution of B to C is represented by 0.19, obtained using the KLEMS dataset during the common subperiod 1970-1996.

5 The Productivity Slowdown

Bruno (1984) suggests that the *productivity slowdown* observed during the seventies might be due to an increase in the prices of raw materials which, as he shows, can be interpreted as a Hicks-neutral negative technological shock on the value added production function.[17] Bruno (1984) shows that the price of raw materials relative to output raised by roughly 30% in the U.S. in 1973. However, raw materials, together with energy, represent only a small part of intermediate goods. Jorgenson and KLEMS datasets provide instead values and prices for the broader set of intermediate goods used in the U.S. economy. With this information at hand, I am able to measure the contribution of intermediate goods to the productivity slowdown using the Cobb-Douglas specification and the CES case presented

[17] Bruno (1984) refers to a "value added bias" as referring to the mismeasurment in value added productivity due to changes in the price of intermediate goods. I choose not to use this terminology here.

above. The exercise proposed recalls the original idea in Bruno (1984) that the productivity slowdown was due to a raise in the intermediate goods price. The main differences here are that I use data for the whole economy, and not only for the manufacturing sector, and the dataset used covers a longer time span.[18] Bruno's (1984) main focus is to estimate the shift in the factor price frontier after the increase in the price of intermediate goods. Here I focus on the importance of the share of intermediate goods in determining the slowdown in the TFP level.

Table 4 (The Productivity Slowdown)

Cobb-Douglas Case

	(1)	(2)	(3)	Slowdown	Slowdown
Jorgenson	1958-1996	1970-1996	1973-1982	[Difference (3)-(1)]	[Difference (3)-(2)]
C (Solow)	0.89%	0.60%	-0.10%	-0.99%	-0.70%
	(0.87%)	(0.56%)	(0.42%)	(-0.45%)	(-0.14%)
B (*p*=1)	0.24%	0.17%	-0.43%	-0.67%	-0.60%
	(0.27%)	(0.29%)	(-0.08%)	(-0.35%)	(-0.37%)
B (variable *p*)	0.22%	0.09%	-1.04%	-1.26%	-1.13%
	(0.27%)	(0.25%)	(-0.42%)	(-0.69%)	(-0.67%)
KLEMS	(1)	(2)	(3)	Slowdown	Slowdown
	1970-2004	1970-1996	1973-1982	[Difference (3)-(1)]	[Difference (3)-(2)]
C (Solow)	0.80%	0.69%	0.01%	-0.79%	-0.68%
	(0.70%)	(0.62%)	(0.52%)	(-0.18%)	(-0.10%)
B (*p*=1)	0.23%	0.16%	-0.50%	-0.73%	-0.66%
	(0.26%)	(0.24%)	(-0.14%)	(-0.40%)	(-0.38%)
B (variable *p*)	0.36%	0.21%	-1.28%	-1.64%	-1.49%
	(0.38%)	(0.28%)	(-0.52%)	(-0.90%)	(-0.80%)

The first three columns report the average growth rate for the Solow residual and for B, both for a constant and a variable price of intermediate goods. Each column represents a different subsample. The fourth and the fifth column represent the difference between the third and the first and between the third and the second, respectively.
The corresponding results using the HP trend series are reported in parenthesis.
Source: D. Jorgenson webpage, KLEMS dataset and own calculation.

[18] Bruno's (1984) dataset covers the 1955-1980 period.

Table 4 reports the growth rate of C and B, as defined in (6) and (7) for the whole sample of both datasets and for two sub-samples, 1970-1996 and 1973-1982. The fourth column reports the difference between the growth rate during the slowdown period 1973-1982 and the whole sample and the fifth the difference between the slowdown period and the common 1970-1996 period. Note that from formula (6) the growth rate of C, x_c, is given by the sum of the growth rates of $A^{\frac{1}{\alpha+\beta}}$, x_A, and that of B, x_B,

$$x_C = x_A + x_B. \tag{14}$$

The fourth and fifth columns of Table 4 show that most of the slowdown in C can be accounted for by the slowdown in B. In particular, the case with variable p is the relevant one. As implied by (10), a positive change in p has a negative impact on B. During the slowdown period p increased remarkably, reinforcing the negative effect on B. The change in the growth rate of B is able to explain a slowdown even larger than that occurred to C, in the case of a variable p, in both datasets. This implies that the growth rate of $A^{\frac{1}{\alpha+\beta}}$ is actually increasing during the "slowdown period". This would imply that the "true" exogenous productivity is actually increasing during that period.[19]

Table 5 reports the CES case. The formula for the growth rate of C is the same as in (14) but now x_A represents the growth rate of A (in the Cobb-Douglas case it represents the growth rate of $A^{\frac{1}{\alpha+\beta}}$). Looking at HP filtered series, the slowdown in x_B is able to explain

[19] Note, however, that fixed A, there is a positive effect on $A^{\frac{1}{\alpha+\beta}}$ given by the decrease in the value added share $\alpha+\beta$.

much more than the slowdown in x_C when using the values of the elasticity of substitution suggested by Bruno (1984) and Rotemberg and Woodford (1996). Again, this implies that the exogenous productivity A increases during the "slowdown period".

Table 5 (The Productivity Slowdown)

CES Case

Jorgenson

	(1) 1958-1996	(2) 1970-1996	(3) 1973-1982	Slowdown [Difference (3)-(1)]	Slowdown [Difference (3)-(2)]
C (Solow)	0.89%	0.60%	-0.10%	-0.99%	-0.70%
	(0.87%)	(0.56%)	(0.42%)	(-0.45%)	(-0.14%)
B (KLEMS)	0.20%	0.14%	-0.37%	-0.57%	-0.51%
	(0.22%)	(0.24%)	(-0.07%)	(-0.29%)	(-0.31%)
B (Bruno)	0.31%	0.22%	-0.58%	-0.89%	-0.80%
	(0.35%)	(0.38%)	(-0.11%)	(-0.46%)	(-0.49%)
B (Rotemberg and Woodford)	0.54%	0.39%	-1.00%	-1.54%	-1.39%
	(0.61%)	(0.66%)	(-0.19%)	(-0.80%)	(-0.85%)

KLEMS

	(1) 1970-2004	(2) 1970-1996	(3) 1973-1982	Slowdown [Difference (3)-(1)]	Slowdown [Difference (3)-(2)]
C (Solow)	0.80%	0.69%	0.01%	-0.79%	-0.68%
	(0.70%)	(0.62%)	(0.52%)	(-0.18%)	(-0.10%)
B (KLEMS)	0.18%	0.13%	-0.41%	-0.59%	-0.54%
	(0.20%)	(0.19%)	(-0.11%)	(-0.31%)	(-0.31%)
B (Bruno)	0.28%	0.20%	-0.64%	-0.92%	-0.84%
	(0.32%)	(0.30%)	(-0.17%)	(-0.49%)	(-0.47%)
B (Rotemberg and Woodford)	0.49%	0.35%	-1.12%	-1.61%	-1.47%
	(0.56%)	(0.52%)	(-0.29%)	(-0.85%)	(-0.81%)

The first three columns report the average growth rate for the Solow residual and for B. The three measures of *B* differ in the value of the elasticity of substitution used: 0.14 (KLEMS), 0.45 (Bruno) and 0.686 (Rotemberg and Woodford). Each column represents a different subsample. The fourth and the fifth column report the difference between the third and the first and between the third and the second, respectively.
The corresponding results using the HP trend series are reported in parenthesis.
Source: D. Jorgenson webpage, KLEMS dataset and own calculation.

Both production specifications are then able to show that the variation in the IGS, together with the change in the price of intermediate goods in the Cobb-Douglas case, is capable to account for the productivity slowdown occurred during the seventies. In some cases, the implied exogenous productivity A displays higher growth during the slowdown period.

The intermediate goods explanation, except for Bruno (1984) and Baily (1986) for the manufacturing sector, has not been given serious consideration in the literature. The argument made in this paper instead clarifies that the evolution of the share of intermediate goods has important implications for our understanding of the growth process.

6 Decompositions

To give some additional insights on the reasons underlying the behavior of the IGS, I perform two decompositions in this section. For this purpose I use Jorgenson's dataset, which covers a longer timer span with respect to KLEMS. The IGS can be defined as

$$IGS_t = \sum_{i=1}^{35} IGS_t^i \varphi_t^i, \tag{15}$$

where $IGS_t^i \equiv p_{it} M_{it} / G_{it}$ is the intermediate goods share in sector i at t and $\varphi_t^i \equiv G_{it}/G_t$ is the value of gross output in sector i at t over the total value of gross output at t.[20] It follows that changes over time in the IGS might be due to changes in IGS_t^i, in φ_t^i, or in both. The former represents the incidence of intermediate goods in gross output of sector i while the latter can be interpreted as the importance of sector i in the economy. The first decomposition consists in fixing φ_t^i at its 1958 level for all i. I then compute

$$\widetilde{IGS}_t = \sum_{i=1}^{35} IGS_t^i \varphi_{1958}^i. \tag{16}$$

[20]The number of sectors in (15) is that of Jorgenson dataset.

This series is labelled IGS-Decomposition 1 in Figure 4 (dotted line). Next, I fix IGS_t^i at its 1958 level for all i and compute

$$\widehat{IGS}_t = \sum_{i=1}^{35} IGS_{1958}^i \varphi_t^i. \tag{17}$$

This series is labelled IGS-Decomposition 2 in Figure 4 (dashed line).

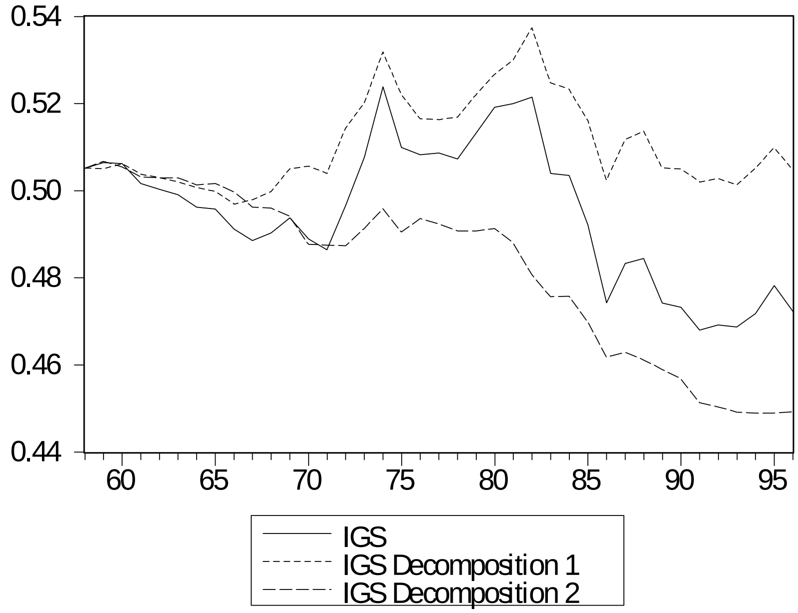

Figure 5: IGS-Decompositions. Source: D. Jorgenson webpage and own calculation.

The IGS-Decomposition 1 presents two remarkable features: the series does not show a decreasing pattern over time while it displays two marked peaks during the oil shocks of the seventies. Apart from the first three years of the sample, the $\widetilde{IGS}_t$ series lies above the actual IGS and appears to drive the short run variability but not the secular trend of the latter.

On the other hand, the IGS-Decomposition 2 decreases steadily over time. The decreasing pattern observed in the IGS appears to be the result of sectorial reallocations. Changes in the importance of the various sectors in producing gross output drive the IGS decline: the weight of those sectors with an high IGS^i_{1958} decrease over time with respect to sectors that had a low IGS^i_{1958}. This suggests that sectorial reallocation might in part be driven by intermediate goods usage.

The exercises proposed in this section are quite instructive on the elements determining TFP growth and, in turn, the growth process. As shown in the models above, the IGS is responsible for an important part of what is measured as capital and labor TFP. At the same time, the decompositions proposed show that most of the decrease in the IGS is due to sectorial reallocation. It is easy to imagine that sectors experiencing a stronger decrease in their intermediate goods share face also, because of this fact, a larger increase in capital and labor productivity. Higher productivity in these sectors attracts more capital and labor, increasing the importance of the same sectors in the economy. At an aggregate level, measured TFP increases because sectors with higher productivity, and a smaller intermediate goods share, are growing.

The above reasoning suggests that the observed growth in TFP should not be considered as an exogenous process that makes each sector increase its own productivity but rather as a shift in the sectorial allocation of capital and labor from less productive activities to new, more productive, processes. Thus, a successful theory of TFP should be constructed by taking into account how intermediate goods utilization influence sectorial productivity and, in turn, the allocation of capital and labor which determines aggregate productivity.

7 Conclusions

In this paper, I measure how the decline in the share of intermediate goods in gross output (IGS) affects common measures of total factor productivity (TFP). The IGS declined in the U.S. over the 1958-2004 period. I show that this decline contributes significantly to measured TFP growth during this period, ranging from around 1/5 to almost 2/3 across different specifications considered.

It follows that actual TFP growth is smaller than the one measured using a standard Cobb-Douglas production function in capital and labor only. This is important since the common view is that TFP growth represents roughly two thirds of output growth.[21] Furthermore, I show that the productivity slowdown occurred during the seventies can be explained by accounting for intermediate goods. In some specifications the increase in the IGS accounts for more than the observed productivity slowdown, implying that the exogenous part of productivity has been increasing during the slowdown period. These results ask for an explanation of an increase in exogenous productivity during the seventies and not of a decrease.

A simple decomposition shows that the decline in the IGS appears to be driven by sectorial reallocation in the economy and not by a reduction in the share of intermediate goods in each sector. This fact suggests that it is crucial to understand sectorial allocations

[21] See Cooley and Prescott (1995).

of capital and labor to have a satisfactory theory of TFP.

The accounting exercise in this paper relies on a simple gross output production model, where the intermediate goods sector is not explicitly modelled. In general equilibrium, changes in the price of intermediate goods must reflect some change in the production technology of those goods with respect to that of gross output. The main contribution of this work is to show the quantitative relevance of the variation in the IGS. The results reported call for a theory that can explain the behavior of the intermediate goods share over time. In constructing this theory, one can easily nest the model presented above into a general equilibrium framework. While the current paper studies how the decline in the IGS affects the evolution of TFP in the U.S., a similar mechanism can also play a role in accounting for cross-country differences in TFP. In recent work, Jones (2007) exploits the fact that intermediate goods provide a multiplier effect on exogenous productivity to explain TFP differences across countries.[22]

I conclude with two possible explanations for the decline in the intermediate goods share that may represent possible lines for further research. Imagine that the economy is composed by a number of goods which are both used as final goods and as inputs in the production of all other goods in the economy. In perfect competition, a technological advance in one

[22] This effect appears also in the Cobb-Douglas model presented here through the term $A^{\frac{1}{\alpha+\beta}}$ in equation (6). However, this effect disappears in the CES case presented here. Although the two papers are in the same spirit, they exploit completely different mechanisms. Jones (2007) studies a level effect provided by the share of intermediate goods. Here I study the effect that the rate of change of the IGS, and in particular its decline, has on the evolution of TFP in the U.S. in the last 46 years.

sector represents a decrease in the price of that good for the other sectors. Then, all sectors experience a decrease in the price of that input. If the technological advance occurs in all sectors, the aggregate productivity effect will be higher than the sum of the sector specific technological shocks. This has been shown in Hulten (1978). The intuition is that when a technological shock in one sector occurs, this does not affect only that particular sector but the entire economy through an input-output structure. Sectorial shocks are easier to explain and to motivate than aggregate shocks. Then, increasing TFP in each sector implies that intermediate goods become less expensive as time passes. If the elasticity of substitution between intermediate goods and value added (capital and labor) is sufficiently small, we observe a constant reduction in the intermediate goods share over time. This would imply an additional source of productivity growth in value added as shown in this paper.

Another possible explanation is the energy-saving argument given in Alpanda and Peralta-Alva (2006). In their model, firms invest in new energy-saving technologies after the oil shocks occurred, because the price of energy is expected to continue raising in the future. Energy represents only a small part of marginal cost for the representative firm. However, as it enters the production of all goods, the effect of energy-saving technologies can be reflected in a decrease in the price of intermediate goods once the oil shocks have passed. This explanation would be consistent with the hump-shaped pattern of the IGS during the seventies, but would not explain why the IGS decreases in the period before 1973.

Appendix A: Data Description

Jorgenson's dataset reports, for each sector, the value and the price of four inputs (capital, labor, energy and intermediate goods) and the value and price of output. Values are in millions of current dollars and prices are normalized to 1 in 1992. The dataset covers 35 sectors roughly at the 2-digit SIC level from 1958 to 1996. Variables are defined as: v_k = *value of capital services*, p_k = *price of capital services*, v_l = *value of labor inputs*, p_l = *price of labor inputs*, v_e = *value of energy inputs*, p_e = *price of energy inputs*, v_m = *value of intermediate goods inputs* and p_m = *price of intermediate goods inputs*.

For gross output, two prices are reported: the one received by producers and the one paid by consumers. These are: p^p = *price of output that producers receive*, p^c = *price of output that consumers pay*. The quantity of gross output, q, is then $q = (v_k + v_l + v_e + v_m)/p^p = (v_k + v_l + v_e + v_m + v_T)/p^c$ where v_T is the value of taxes paid by each sector.

The EU KLEMS Database, March 2007, reports a larger number of series with respect to Jorgenson. For this reason I will list only the series used in the paper. These are *GO*, aggregate gross output at current basic prices (millions of U.S. dollars), *II*, aggregate intermediate inputs at current purchasers' prices (millions of US dollars), *GO_P*, gross output price index (normalized to 100 in 1995), *II_P*, intermediate inputs price index (1995=100), *CAP_QI*, capital services volume indices (1995=100), *LAB_QI*, labor services volume indices (1995=100), *CAP*, capital compensation (millions of U.S. dollars), *LAB*, labor compensation (millions of U.S. dollars). The data used to estimate the elasticity of substitution between value added and intermediate goods are described in Appendix D.

Appendix B: Price and Quantity Indices

In this appendix I report the methodology used to obtain the numerical results.

With Jorgenson's dataset I am able to calculate the factor shares of gross output for the entire economy using the formula

$$Share\ of\ input\ j\ at\ t = \frac{\sum_i v_{jit}}{\sum_i p^p_{it} q_{it}}, \tag{18}$$

where $j = k, l, e, m$, and $i = 1, ..., 35$. As I include energy in the definition of intermediate goods, the IGS in Figure 1 is then calculated as

$$IGS_t = \frac{\sum_i v_{mit} + \sum_i v_{eit}}{\sum_i p^p_{it} q_{it}}. \tag{19}$$

The Solow residual average growth rate in Table 1, C, is constructed in the following way. First, I construct a Laspeyres price index for capital and labor using Jorgenson's dataset. The formula for capital input is

$$P^{lasp}_{kt} = \frac{\sum_i p_{kit}(v_{ki1958}/p_{ki1958})}{\sum_i p_{ki1958}(v_{ki1958}/p_{ki1958})}, \tag{20}$$

where p_{kit} is the price of capital input in sector i at time t and v_{kit} is the value of capital input in sector i at time t. Consequently, p_{ki1958} and v_{ki1958} are the price and the value of capital in sector i in the base year 1958. Clearly, v_{ki1958}/p_{ki1958} represents the quantity of capital used in sector i in 1958. The price index for labor, P^{lasp}_{lt}, is constructed in the same fashion using p_{lit}, v_{lit}, p_{li1958} and v_{li1958} instead of the corresponding variables for capital.

Then I construct Paasche price indices for capital and labor inputs. The price index for capital input is

$$P_{kt}^{paas} = \frac{\sum_i v_{kit}}{\sum_i p_{ki1958}\left(v_{kit}/p_{kit}\right)}. \tag{21}$$

Here v_{kit}/p_{kit} represents the quantity of capital used in sector i at time t. The price index for labor is constructed accordingly.

With Laspeyres and Paasche indices, I construct the Fisher index as

$$P^{fisher} = \sqrt{P^{lasp} P^{paas}}. \tag{22}$$

The Fisher index is constructed for both capital and labor. I use P_{kt}^{fisher} and P_{lt}^{fisher} to deflate the total value of capital and labor inputs in the economy. That is, the index for real capital is found as

$$K_t = \frac{\sum_i v_{kit}}{P_{kt}^{fisher}}. \tag{23}$$

In a similar fashion, using the labor price index, I find the index for real labor L_t. The Solow residual is then constructed as

$$C_t = \frac{Y_t}{K_t^{\gamma} L_t^{1-\gamma}}, \tag{24}$$

where Y_t is the real GDP series from the U.S. Bureau of Economic Analysis and $\gamma = 0.3254$ is computed as the average capital share of primary inputs (capital and labor) over the period covered in Jorgenson's dataset. The average growth rate μ is then found from the growth factor of C_t over the entire period, $1 + x$, as

$$\mu = (1 + x)^{1/(T-1)} - 1 \tag{25}$$

where T is the length of the period considered.

With the KLEMS dataset, the Solow residual is also constructed using (24). The capital and labor series are CAP_QI and LAB_QI, capital and labor services. The parameter γ is constructed as the ratio of capital compensation to capital and labor compensation, $CAP/(CAP+LAB)=0.3339$. The series for Y_t is the real GDP series from the U.S. Bureau of Economic Analysis. The average growth rate is then computed using (25).

With Jorgenson's data, B_t in Table 1 is constructed plugging (19) into formula (10) and setting $p_t = 1 \ \forall \ t$, and then computing the average growth rate of this series using the equivalent of formula (25). With KLEMS I adopt the same procedure. The intermediate goods share is given by II/GO.

Again, with Jorgenson's data the B_t in Table 2 is constructed using first formula (10) for the level of B_t. The relative price p_t is in this case computed as the ratio of an intermediate goods Fisher price index and a gross output Fisher price index. I apply the equivalent to formulas (20) and (21) to construct the price index in (22), named P_{out}^{fisher}, for gross output. As pointed out above, Jorgenson reports series for both intermediate goods and energy while the definition of intermediate goods I employ includes both categories. I then construct the Laspeyres and Paasche price indices using both Jorgenson's intermediate goods and energy series. From these I construct the Fisher index for intermediate goods, P_{int}^{fisher}. Then, I take the ratio $P_{int}^{fisher}/P_{out}^{fisher}$ to obtain the Fisher relative price p_t reported in Figure 3 and used to construct the average growth rate of B_t appearing in Table 2. The latter is then computed as in the case with $p_t = 1$. With KLEMS data the relative price of intermediate goods is given by II_P/GO_P. The average growth rate of B_t is then computed as with Jorgenson's data.

Appendix C: The CES Case

In this appendix I report the calculations performed to obtain the reduced form problem (12).

From the gross output maximization problem

$$\max_{K,N,M}\left\{A\left[\theta M^{\sigma}+(1-\theta)\left(K^{\gamma}N^{1-\gamma}\right)^{\sigma}\right]^{1/\sigma}-pM-rK-wN\right\}, \tag{26}$$

I compute the first order condition with respect to M. This can be written as

$$\frac{G}{\left[\theta M^{\sigma}+(1-\theta)\left(K^{\gamma}N^{1-\gamma}\right)^{\sigma}\right]}\theta M^{\sigma}=pM.$$

Since $\frac{G^{\sigma}}{A^{\sigma}}=\left[\theta M^{\sigma}+(1-\theta)\left(K^{\gamma}N^{1-\gamma}\right)^{\sigma}\right]$, the above expression can be written as

$$\theta M^{\sigma}=\frac{pM}{G}\frac{G^{\sigma}}{A^{\sigma}}.$$

I use the last expression to substitute for θM^{σ} in the gross output production function G. This becomes

$$G=A\left[\frac{pM}{G}\frac{G^{\sigma}}{A^{\sigma}}+(1-\theta)\left(K^{\gamma}N^{1-\gamma}\right)^{\sigma}\right]^{1/\sigma}$$

and, after some algebra it can be written

$$G = \frac{A(1-\theta)^{1/\sigma}}{\left[1-\frac{pM}{G}\right]^{1/\sigma}} K^{\gamma} N^{1-\gamma}. \tag{27}$$

Equation (27) expresses gross output production as a function of the level of productivity A, the share of intermediate goods in gross output production $\frac{pM}{G}$ and the level of value added. This expression holds in equilibrium. I plug (27) in problem (26), and use the equality $pM = \frac{pM}{G}G$ to write

$$\begin{aligned}
&\max_{K,N}\left\{G - \frac{pM}{G}G - rK - wN\right\} \equiv \\
&\equiv \max_{K,N}\left\{\frac{A(1-\theta)^{1/\sigma}}{\left[1-\frac{pM}{G}\right]^{1/\sigma}} K^{\gamma} N^{1-\gamma} - \frac{pM}{G}\frac{A(1-\theta)^{1/\sigma}}{\left[1-\frac{pM}{G}\right]^{1/\sigma}} K^{\gamma} N^{1-\gamma} - rK - wN\right\} \\
&\equiv \max_{K,N}\left\{\left[1-\frac{pM}{G}\right]\frac{A(1-\theta)^{1/\sigma}}{\left[1-\frac{pM}{G}\right]^{1/\sigma}} K^{\gamma} N^{1-\gamma} - rK - wN\right\} \\
&\equiv \max_{K,N}\left\{\left[1-\frac{pM}{G}\right]^{1-1/\sigma} A(1-\theta)^{1/\sigma} K^{\gamma} N^{1-\gamma} - rK - wN\right\} \qquad (28)
\end{aligned}$$

The level of productivity in problem (28) depends now on $\left[1-\frac{pM}{G}\right]^{1-1/\sigma}$, which is a function of the IGS, $\frac{pM}{G}$. It is clear that a decrease in the IGS acts as a positive technological shock if $\sigma < 0$ and a negative one if $0 < \sigma \leq 1$.

Appendix D: Estimating the Elasticity of Substitution

The elasticity of substitution between intermediate goods and value added is defined as

$$\varepsilon = -\frac{d(V/M)}{d(p_v/p_m)}\frac{p_v/p_m}{V/M} \tag{29}$$

Let $x = V/M$ and $y = p_v/p_m$. Then (29) reads as

$$\varepsilon = -\frac{dx}{dy}\frac{y}{x} = -\frac{d\log x}{d\log y} \tag{30}$$

Substitute for x and y their original definitions to obtain

$$\varepsilon = -\frac{d\log V - d\log M}{d\log p_v - d\log p_m} \tag{31}$$

The differentials can be approximated by discrete variations as

$$\varepsilon = -\frac{\Delta v - \Delta m}{\Delta \tilde{p}_v - \Delta \tilde{p}_m} \tag{32}$$

where $v = \log V$, $m = \log M$, $\tilde{p}_v = \log p_v$, $\tilde{p}_m = \log p_m$. Equation (32) can be estimated in the form

$$\Delta v - \Delta m = -\varepsilon \left(\Delta \tilde{p}_v - \Delta \tilde{p}_m\right). \tag{33}$$

To estimate (33) I use data of indices of volume and price of value added and intermediate goods from the EU KLEMS Project for 54 sectors of the U.S. economy. As I am interested in one common elasticity of substitution I estimate a pooled OLS of equation (33).[23] The estimate of $-\varepsilon$ in equation (33) is -0.1392 with a standard error of 0.01757, implying that the estimation is significantly different from zero. The CES production function implies $\varepsilon = 1/(1-\sigma)$, so that the from the estimation I recover $\sigma = -6.18$.

The 54 sectors used are: Agriculture; Forestry; Fishing; Mining of coal and lignite and extraction of peat; Extraction of crude petroleum and natural gas and services; Mining of metal ores; Other mining and quarrying; Food and beverages; Tobacco; Textiles; Wearing apparel, dressing and dying of fur; Leather, leather and footwear; Wood and of Wood and Cork; Pulp and paper; Printing, publishing and reproduction; Coke, refined petroleum and

[23] A similar methodology is used in Rotemberg and Woodford (1996).

nuclear fuel; Chemicals; Rubber and plastics; Other non-metallic minerals; Basic metals; Fabricated metal; Machinery, nec; Office, accounting and computing machinery; Electrical machinery and apparatus, nec; Radio, television and communication equipment; Medical, precision and optical instruments; Motor vehicles, trailers and semi-trailers; Other transport equipment; Manufacturing, nec; Electricity and Gas; Construction; Sale, maintenance, repair of motor vehicles and motorcycles and retail sale of fuel; Wholesale trade and commission trade, except of motor vehicles and motorcycles; Retail trade, except of motor vehicles and motorcycles, repair of household goods; Hotels and Restaurants; Inland transport; Water transport; Air transport; Supporting and auxiliary transport activities and activities of travel agencies; Post and Telecommunications; Financial intermediation, except insurance and pension funding; Insurance and pension funding, except compulsory social security; Real estate activities; Renting of machinery and equipment; Computer and related activities; Research and development; Other business activities; Public administration and defence, and compulsory social security; Education; Health and social work; Sewage and refuse disposal, sanitation and similar activities; Activities of membership organizations, nec; Recreational, cultural and sporting activities; Other service activities.

References

[1] Alpanda, S. and Peralta-Alva, A., (2006), "Oil Crisis, Energy-Saving Technological Change and the Stock Market Crash of 1973-74", mimeo.

[2] Baily, M., (1986), "Productivity and Materials Use in U.S. Manufacturing", *Quarterly Journal of Economics*, Vol. 101, No. 1, pp. 185-196.

[3] Bruno, M., (1984), "Raw Materials, Profits and the Productivity Slowdown", *Quarterly Journal of Economics*, Vol.99, No. 1, pp. 1-30.

[4] Cooley, F.C., and Prescott, E.C., (1995), "Economic Growth and Business Cycles", in T. F. Cooley *Frontiers of Business Cycle Research*, Princeton University Press.

[5] Castro, R., Clementi, G. L., and MacDonald, G., (2006), "Legal Institutions, Sectoral Heterogeneity, and Economic Development", SSRN working paper http://ssrn.com/abstract=869755.

[6] Guner, N., Ventura, G. and Xu, Y., (2007), "Macroeconomic Implications of Size Dependent Policies", CEPR Discussion Paper No. 6138.

[7] Herrendorf, B. and Teixeira, A., (2005), "How Barriers to International Trade Affect TFP", *Review of Economic Dynamics*, Vol. 8, No. 4, pp. 866-876.

[8] Hulten, C.R., (1978), "Growth Accounting with Intermediate Goods", *Review of Economic Studies*, Vol 45, No.3, pp. 511-518.

[9] Jones, C. I., (2007), "Intermediate Goods and Weak Links: A Theory of Economic Development", mimeo.

[10] Jorgenson, D. W., Gollop, F. M. and Fraumeni B. M. (1987), "Productivity and U.S. Economic Growth", Harvard University Press, Cambridge, MA.

[11] Lagos, R., (2006), "A Model of TFP", *Review of Economic Studies* 73, 983–1007.

[12] Parente, S. and Prescott, E. C., (1999), "Monopoly Rights: A Barrier to Riches", *American Economic Review*, Vol. 89, No. 5, pp. 1216-1233.

[13] Prescott, E.C., (1998), "Lawrence R. Klein Lecture 1997: Needed: A Theory of Total Factor Productivity", *Internation Economic Review*, Vol. 39, No. 3., pp. 525-551.

[14] Restuccia, D. and Rogerson, R., (2007), "Policy Distortions and Aggregate Productivity with Heterogeneous Plants", NBER Working Paper No. 13018.

[15] Rotemberg, J.J. and Woodford, M., (1995), "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets", in T. F. Cooley *Frontiers of Business Cycle Research*, Princeton University Press.

[16] Rotemberg, J.J. and Woodford, M., (1996), "Imperfect Competition and the Effects of Energy Price Increases on Economic Activity", *Journal of Money, Credit & Banking*, Vol. 28.

[17] Wolff, N. W., (1996), "The Productivity Slowdown: Culprit at Last? Follow-up on Hulten and Wolff", *American Economic Review*, Vol.86, No. 5, pp.1239-1252.